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10MAT31

Third Semester B.E. Degree Examination, December 2012
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Find the Fourier series of $f(x) = x - x^2$, $-\pi \leq x \leq \pi$. Hence deduce that
- $$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$
- (07 Marks)

Is the above deduced series convergent? (Answer in Yes or No)

- b. Define : i) Half range Fourier sine series of $f(x)$
 ii) Complex form of Fourier series of $f(x)$
 Find the half range cosine series of $f(x) = x$ in $0 < x < 2$. (07 Marks)
- c. Obtain a_0 , a_1 , b_1 in the Fourier expansion of y , using harmonic analysis for the data given.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(06 Marks)

- 2 a. Find the Fourier transform of
- $$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$
- Hence evaluate $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$ (07 Marks)
- b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ (07 Marks)
- c. Find the Fourier cosine transform of
- $$f(x) = \begin{cases} 4x & , \text{ for } 0 < x < 1 \\ 4 - x & , \text{ for } 1 < x < 4 \\ 0 & , \text{ for } x > 4 \end{cases}$$
- (06 Marks)

- 3 a. i) Write down the two dimensional heat flow equation (p d e) in steady state (or two dimensional) Laplace's equation. Just mention.
 ii) Solve one dimensional heat equation by the method of separation of variables. (07 Marks)
- b. Using D'Alembert's method, solve one dimensional wave equation. (07 Marks)
- c. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form of $y = a \sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is,

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$$

Start the answer assuming the solution to be

$$y = (C_1 \cos(px) + C_2 \sin(px))(C_3 \cos(cpt) + C_4 \sin(cpt))$$

(06 Marks)

- 4 a. Fit a linear law, $P = mW + C$, using the data

P	12	15	21	25
W	50	70	100	120

(06 Marks)

- b. Find the best values of a and b by fitting the law $V = at^b$ using method of least squares for the data,

V (ft/min)	350	400	500	600
t (min)	61	26	7	26

Use base 10 for algorithm for computation.

(07 Marks)

- c. Using simplex method,

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{Subject to, } x_1 + x_2 \leq 2 ; 5x_1 + 2x_2 \leq 10 ; 3x_1 + 8x_2 \leq 12 ; x_1, x_2 \geq 0.$$

(07 Marks)

PART – B

- 5 a. Use Newton-Raphson method, to find the real root of the equation $3x = (\cos x) + 1$.

Take $x_0 = 0.6$. Perform two iterations.

(06 Marks)

- b. Apply Gauss-Seidel iteration method to solve equations

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3z + 20z = 25$$

Assume initial approximation to be $x = y = z = 0$. Perform three iterations.

(07 Marks)

- c. Using Rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Take $[1 \ 0 \ 0]^T$ as the initial approximation. Perform four iterations.

(07 Marks)

- 6 a. Use appropriate interpolating formula to compute $y(82)$ and $y(98)$ for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

(07 Marks)

- b. i) For the points (x_0, y_0) (x_1, y_1) (x_2, y_2) mention Lagrange's interpolation formula.

- ii) If $f(1) = 4$, $f(3) = 32$, $f(4) = 55$, $f(6) = 119$; find interpolating polynomial by Newton's divided difference formula.

(07 Marks)

- c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$, using

- i) Simpson's $1/3^{\text{rd}}$ rule ii) Simpson's $3/8^{\text{th}}$ rule iii) Weddle's rule, using

x	0	1	2	3	4	5	6
$f(x) = \frac{1}{1+x^2}$	1	0.5	0.2	0.4	0.0588	0.0385	0.027

(06 Marks)

- 7 a. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t), u(4, t) = 0$. $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking $h = 1, k = 0.5$ upto four steps. (07 Marks)
- b. Solve two dimensional Laplace equation at the pivotal or nodal points of the mesh shown in Fig.Q7(b). To find the initial values assume $u_4 = 0$. Perform three iterations including computation of initial values. (07 Marks)

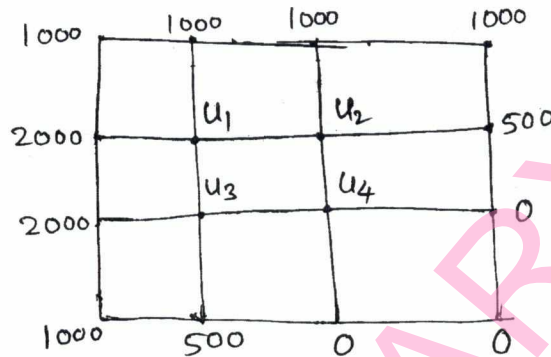


Fig.Q7(b)

- c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions $u(x, 0) = \sin \pi x, 0 \leq x \leq 1$; $u(0, t) = u(1, t) = 0$. Carry out computations for two levels, taking $h = 1/3, k = 1/36$. (06 Marks)

- 8 a. Find the z-transform of

$$\frac{n}{3^n} + 2^n n^2 + 4 \cos(n\theta) + 4^n + 8 \quad (07 \text{ Marks})$$

- b. State and prove i) Initial value theorem ii) Final value theorem of z-transforms. (07 Marks)

- c. Using the z-transform solve

$$u_{n+2} + 4u_{n+1} + 3u_n = 3^n \text{ with } u_0 = 0, u_1 = 1. \quad (06 \text{ Marks})$$

Third Semester B.E. Degree Examination, December 2012
Electronic Circuits

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Explain transistor in its fixed bias mode with relevant expression. (06 Marks)
 b. With a neat sketch, explain transistor as a switch. (06 Marks)
 c. For the circuit shown calculate I_B , I_C , V_{CE} , V_C , V_E , V_B . Assume $\beta = 100$. (08 Marks)

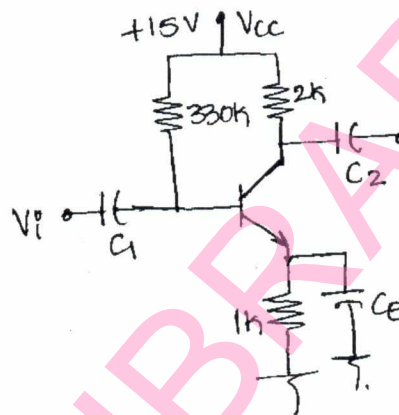


Fig.Q.1(c)

- 2 a. Explain the VI characteristics of n-channel JFET and define various conditions. (08 Marks)
 b. Explain the construction and working of n-channel depletion mode MOSFET. (08 Marks)
 c. Mention merits and demerits of IGBT. (04 Marks)
- 3 a. Explain the construction and working of phototransistor and mention its applications. (10 Marks)
 b. What are optocouplers? Explain the working and characteristics of optocoupler. (10 Marks)
- 4 a. Derive expression for A_i , Z_i , A_v , Y_o , A_p for a transistor amplifier using h-parameter model. (12 Marks)
 b. Explain the need for cascading amplifier and with the block diagram, explain two stage cascaded amplifier. (08 Marks)

PART – B

- 5 a. Explain different fb amplifiers. (08 Marks)
 b. With the block diagram, explain the negative feedback in small signal amplifier. (06 Marks)
 c. An amplifier having a voltage gain of 60dB uses $1/20^{\text{th}}$ of its output in negative feedback. Calculate the gain with feedback, the percentage change in gain without and with feedback consequent on 50% change in gm. (06 Marks)

- 6 a. Explain the construction and working of RC phase shift oscillator. (08 Marks)
b. Find the frequency of the oscillations of a Colpitts oscillator having $C_1 = 150\text{pF}$, $C_2 = 1.5\text{ nF}$ and $L = 50\mu\text{H}$. (04 Marks)
c. With a circuit diagram, explain the working of RC low pass and RC high pass circuits. (08 Marks)
- 7 a. With a block diagram, explain the working of three terminal voltage regulators. (06 Marks)
b. Explain the construction and working of SMPS and mention different types of switching regulators. (08 Marks)
c. Define the terms load regulation, line regulation and output resistance for a voltage regulator. (06 Marks)
- 8 a. Briefly explain characteristics of an ideal op-amp and compare with practical op-amp. (10 Marks)
b. With relevant formulas, neat diagram and wave form explain op-amp Schmitt trigger. (10 Marks)

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Third Semester B.E. Degree Examination, December 2012
Logic Design

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Differentiate analog and digital signals. Define period, frequency and duty cycle of a digital signal. Prove that the duty cycle of a symmetrical waveform is 50%. (06 Marks)
- b. What are universal gates? Realize $((\overline{A+B}) \cdot C)$ using only NAND gates. (05 Marks)
- c. Describe positive and negative logic. List the equivalences between them. (04 Marks)
- d. What is the need for HDL? Explain the structure of VHDL / verilog program. (05 Marks)

- 2 a. Find the minimal SOP of the following Boolean functions using K-Maps:
 - i) $f(a, b, c, d) = \sum m(6, 7, 9, 10, 13) + d(1, 4, 5, 11)$
 - ii) $f(w, x, y, z) = \pi m(1, 2, 3, 4, 9, 10) + d(0, 1, 4, 15)$ (08 Marks)
- b. Simplify $f(A, B, C, D) = \sum m(0, 1, 2, 3, 10, 11, 12, 13, 14, 15)$ using Quine-Mc Clusky method. (08 Marks)
- c. What are static hazards? How to design a hazard free circuit? Explain with an example. (04 Marks)

- 3 a. Design and implement BCD to excess-3 code converter using four 8:1 multiplexers. Take MSB 'A' as map entered variable (input variable) 'BCD' lines as select lines, assuming $f(A, B, C, D)$ as BCD input. (08 Marks)
- b. Realize a logic circuit for Octal to binary encoder. (06 Marks)
- c. Draw the PLA circuit and realize the Boolean functions: $X = A'B'C + AB'C' + B'C$, $Y = A'B'C + AB'C'$, $Z = B'C$ (04 Marks)
- d. Give the HDL implementation of 2:1 MUX. (02 Marks)

- 4 a. Draw the logic diagram, truth table and timing diagram for edge-triggered D-flip flop. (06 Marks)
- b. With a neat logic diagram and truth table, explain the working of JK Master-Slave Flip-Flop along with its implementation using NAND gates. (06 Marks)
- c. Analyze the behavior of the sequential circuit shown in Fig. Q4 (c) and draw the state table and state transition diagram. (08 Marks)

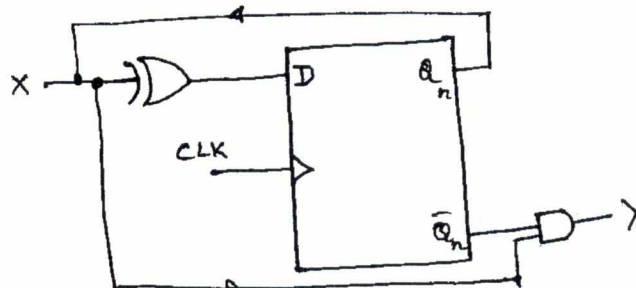


Fig. Q4 (c)

PART – B

- 5 a. Using positive edge triggered D flip-flops, draw the logic diagram for a 4-bit parallel-in-serial-out (PISO) shift register and explain its working to load 1001 into it and shift the same. **(08 Marks)**
- b. With a neat diagram, explain how shift registers can be applied for serial addition. **(05 Marks)**
- c. How long will it take to shift an 4 bit number into 4 bit PISO shift register that operates at clock frequency of 5 MHz. Also, what is the time required to extract 4-bit number from PISO operates at 5 MHz clock? **(04 Marks)**
- d. Write verilog/VHDL code for Johnson counter. **(03 Marks)**
- 6 a. With the help of neat block diagram and timing diagram, explain the working of a Mod-16 ripple counter constructed using positive edge triggered JK flip-flops. **(08 Marks)**
- b. Design a self-correcting Mod-5 synchronous down counter using JK flip-flops. Assume 100 as the next state for all the unused states. **(08 Marks)**
- c. List any two drawbacks of asynchronous counter. What is the clock frequency in a 3-bit counter, if the clock period of the waveform at last flip-flop is 24 μ s? **(04 Marks)**
- 7 a. How does state transition diagram of a Moore machine differ from Mealy machine? Draw the state transition diagram of synchronous sequential logic circuit using Mealy model that detects three consecutive zeros from an input data stream, X and signals detection by making output, Y = 1. **(06 Marks)**
- b. Draw the ASM chart for vending machine problem using Mealy model. **(08 Marks)**
- c. What is the use of state reduction technique? Demonstrate the state reduction by implication table method. **(06 Marks)**
- 8 a. With a neat diagram, explain the working of a 4-bit D/A converter. **(08 Marks)**
- b. What is the accuracy and resolution of an 8-bit D/A converter? Find the resolution and accuracy of the same if the full scale output is +10V. **(04 Marks)**
- c. Discuss the working of following A/D converters:
- Successive approximation A/D.
 - Counter type A/D. **(08 Marks)**

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Third Semester B.E. Degree Examination, December 2012
Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. For any sets A, B, C, D prove by using the laws that

$$(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B) = B$$
 (06 Marks)
 - b. If $S, T \subseteq U$, prove that S and T are disjoint if and only if $S \cup T = S \Delta T$ (04 Marks)
 - c. In a survey of 120 passengers, an airline found that 48 preferred ice cream with their meals, 78 preferred fruits and 66 preferred coffee. In addition, 36 preferred any given pair of these and 24 passengers preferred them all. If two passengers are selected at random from the survey sample of 120, what is the probability that
 - i) they both preferred only coffee with their meals
 - ii) they both preferred exactly two of three offerings. (06 Marks)
 - d. A student visits a sports club everyday from Monday to Friday after school hours and plays one of the three games: Cricket, Tennis and Football. In how many ways can he play each of the three games at least once during a week (from Monday to Friday)? (04 Marks)
- 2 a. Define Tautology. Prove that, for any propositions p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology using truth table. (06 Marks)
 - b. Prove the following logical equivalence without using truth table

$$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$$
 (04 Marks)
 - c. Define the dual of a logical statement. Write down the dual of

$$[(p \vee T_0) \wedge (q \vee F_0)] \vee [(r \wedge s) \wedge T_0]$$
 (04 Marks)
 - d. Simplify the following switching network. [Refer Fig.Q2(d)] (06 Marks)

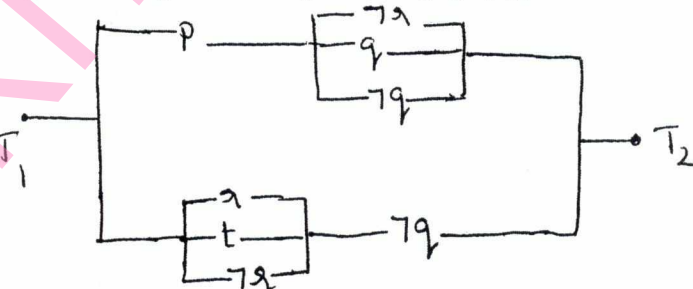


Fig.Q2(d)

- 3 a. If $p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4 = 0$, $s(x): x^2 - 3 > 0$, find the truth values of the following:

i) $\exists x [p(x) \wedge q(x)]$	ii) $\forall x [p(x) \rightarrow q(x)]$	iii) $\forall x [q(x) \rightarrow s(x)]$
iv) $\forall x [r(x) \vee s(x)]$	v) $\exists x, [p(x) \wedge r(x)]$	vii) $\forall x, [r(x) \rightarrow p(x)]$

(06 Marks)

- b. Negate and simplify each of the following:
 i) $\forall x, [p(x) \wedge \neg q(x)]$ ii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$ (04 Marks)
- c. Establish the validity of the following argument:

$$\frac{\forall x, [p(x) \vee q(x)] \quad \forall x, [\{\neg p(x) \wedge q(x)\} \rightarrow r(x)]}{\therefore \forall x, [\neg r(x) \rightarrow p(x)]}$$
 (06 Marks)
- d. Given $R(x, y)$: $x + y$ is even where the variables x and y represent integers write down the following in words:
 i) $\forall x, \exists y p(x, y)$ ii) $\exists x, \forall y p(x, y)$ (04 Marks)
- 4 a. Prove that $4n < (n^2 - 7)$, for all integers $n \geq 6$. (06 Marks)
- b. Obtain a recursive definition for the sequence $\{a_n\}$ in each of the following:
 i) $a_n = 5n$ ii) $a_n = 2 - (-1)^n$ (04 Marks)
- c. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's. (06 Marks)
- d. If F_0, F_1, F_2, \dots are Fibonacci numbers, prove that

$$\sum_{i=0}^n F_i^2 = F_n \times F_{n+1}$$
, for all positive integers n . (04 Marks)

PART - B

- 5 a. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one-to-one and onto. (06 Marks)
- b. Define Stirling number of second kind and evaluate $S(8, 6)$. (04 Marks)
- c. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1$, $g(x) = 3x$,

$$h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$$

 Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$. (06 Marks)
- d. Show that if any $(n + 1)$ numbers from 1 to $2n$ are chosen, then two of them will have their sum equal to $(2n + 1)$. (04 Marks)
- 6 a. Let $A = \{1, 2, 3, 4\}$, $B = \{w, x, y, z\}$ and $C = \{5, 6, 7\}$. Also let R_1 be a relation from A to B , defined by $R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$ and R_2 and R_3 be relations from B and C , defined by $R_2 = \{(w, 5), (x, 6)\}$, $R_3 = \{(w, 5), (w, 6)\}$. Find $R_1 \circ R_2$ and $R_1 \circ R_3$. (06 Marks)
- b. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. On this set define the relation R by $(x, y) \in R$ if and only if $(x - y)$ is a multiple of 5. Verify that R is an equivalence relation. (04 Marks)
- c. Let (A, R_1) and (B, R_2) be posets. On $A \times B$, define the relation R by $(a, b)R(x, y)$ if aR_1x and bR_2y . Prove that R is a partial order. (06 Marks)

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Third Semester B.E. Degree Examination, January 2013
Data Structures with C

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. What are pointer variables? How to declare a pointer variable? (05 Marks)
 b. What are the various memory allocation techniques? Explain how memory can be dynamically allocated using malloc ()? (10 Marks)
 c. What is recursion? What are the various types of recursion? (05 Marks)
- 2 a. What is the difference between int *a and int a[5] and int *[5]? (06 Marks)
 b. What is a structure? How to declare and initialize a structure? (06 Marks)
 c. Write a program in C to read a sparse matrix of integer values and search this matrix for an element specified by the user. (08 Marks)
- 3 a. Define stack. List the operations on stack. (08 Marks)
 b. Obtain the postfix and prefix expression for $((A + (B - C) * D) ^ E) + F$. (06 Marks)
 c. What is system stack? How the control is transferred to or from the function with the help of activation record? (06 Marks)
- 4 a. What is a linked list? Explain the different types of linked list with diagram. (10 Marks)
 b. Write a function to insert a node at front and rear end in a circular linked list. Write down sequence of steps to be followed. (10 Marks)

PART – B

- 5 a. What is a tree? Explain: i) root node, ii) child, iii) siblings, iv) ancestors using structure representation. (06 Marks)
 b. What is a binary tree? How it is represented using array and linklist? (10 Marks)
 c. What is a heap? Explain the different types of heap? (04 Marks)
- 6 a. What is a binary search tree? Draw the binary search tree for the following input: 14, 5, 6, 2, 18, 20, 16, 18, -1, 21. (10 Marks)
 b. What is a forest? Explain the different method of traversing a tree with following tree:

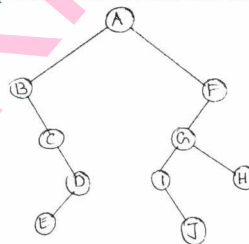


Fig.Q6(b)

- 7 a. What is priority queue? Explain the various types of priority queues. (08 Marks)
 b. Write short notes on: i) Binomial heaps, ii) Fibonacci heap. (06 Marks)
 c. What is leftist tree? Explain different types of leftist trees. (06 Marks)
- 8 a. What is an AVL tree? Write the algorithm to insert an item in to AVL tree. (08 Marks)
 b. Write short notes on: i) Red-Black tree, ii) Splay trees. (06 Marks)
 c. Explain the different types of rotations of an AVL tree. (06 Marks)

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Third Semester B.E. Degree Examination, January 2013
Object Oriented Programming with C++

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
atleast TWO questions from each part.**

PART – A

- 1 a. Explain the following OOP features :
 i) Class ii) Encapsulation iii) Polymorphism iv) Inheritance. (08 Marks)
 b. Define inline function. Explain with an example program. What are the conditions, where inline functions can not be expanded? (06 Marks)
 c. Define function overloading. Demonstrate with C++ program. (06 Marks)

- 2 a. Write a C++ program to keep track of the number of objects created by a particular class without, using extern variable. (06 Marks)
 b. Determine the output for the following snippets and comment. (06 Marks)

i) class A

```

{ int x = 10 ;
void display( )
{
    cout <<"The value of x = " <<x ;
}
};
void main( )
{
    A obj ;
    obj. display( );
}

```

ii) class A

```

{ int pvt ;
Public ; int * ptr – pub ;
A( ) {
    pvt = 25 ;
    pvt_pub = &pvt ;
}
void print_private( )
{
    cont << pvt <<endl ;n
}
};
void main( )
{
    A objA ;
    * objA. ptr_pub = 10 ;
    objA. print_private( )
}.

```

- c. Demonstrate with C++ program for
 i) Passing objects to functions ii) Returning objects. (08 Marks)

- 3 a. Define friend function. Explain with a C++ program to add 2 complex numbers. (08 Marks)
 b. Write a C++ program to overload pre – increment and post – increment operators. (08 Marks)
 c. State the advantages of Generic functions and classes and give the syntax for both.(04 Marks)
- 4 a. Write bubble sort program in C++ to sort int and double data type elements, using template function. (08 Marks)
 b. List out the impacts on public, protected and private data members of base class, when the base class is derived by
 i) public
 ii) protected
 iii) private access specifies. Demonstrate it by writing separate C++ program for each. (12 Marks)

PART – B

- 5 a. What ambiguities arise when multiple base class are inherited. How do you resolve them? Explain with a C++ program. (08 Marks)
 b. Explain with a C++ program how to pass parameters to base – class constructor. (08 Marks)
 c. Discuss the order of invocation of constructor and destructor. (04 Marks)
- 6 a. Show that in C++, virtual functions are hierarchical. (06 Marks)
 b. What is an abstract class? Write a C++ program to implement the abstract class. (10 Marks)
 c. Write a short note on early v/s late binding. (04 Marks)
- 7 a. With examples, list and explain any four I/O manipulators in C++. (06 Marks)
 b. Demonstrate the random access to files in C++, using relevant stream class functions. (06 Marks)
 c. What is the necessity of exception handling? Show how multiple catch statements are used. (08 Marks)
- 8 Write short note on the following
 a. Copy constructor
 b. New and delete
 c. Lists
 d. Vectors. (20 Marks)

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Third Semester B.E. Degree Examination, December 2012

Advanced Mathematics – I

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions.

- 1** a. Find the modulus and amplitude of the complex number $1 - \cos\alpha + i\sin\alpha$. (05 Marks)
 b. If z_1 and z_2 are two complex numbers, show that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\{|z_1|^2 + |z_2|^2\}$. (05 Marks)
 c. Find the fourth roots of $-1 + i\sqrt{3}$. (05 Marks)
 d. If $2\cos\theta = x + \frac{1}{x}$, prove that $2\cos r\theta = x^r + \frac{1}{x^r}$. (05 Marks)
- 2** a. Find the n^{th} derivative of $e^{2x} \cos^3 x$. (07 Marks)
 b. Find the n^{th} derivative of $\frac{x}{x^2 - 5x + 6}$. (06 Marks)
 c. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. (07 Marks)
- 3** a. Find the angle between the pair of curves $r = 6 \cos \theta$, $r = 2(1 + \cos \theta)$. (07 Marks)
 b. Find the pedal equation of the curve $r^2 = a^2 \sin 2\theta$. (06 Marks)
 c. Obtain the Maclaurin's series expansion of the function $\sqrt{1 + \sin 2x}$. (07 Marks)
- 4** a. If $u = x^2y + y^2z + z^2x$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$. (05 Marks)
 b. If $u = \tan^{-1}\left(\frac{x^3y^3}{x^3 + y^3}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2} \sin 2u$. (05 Marks)
 c. If $u = x + y + z$, $v = y + z$, $z = uvw$, find Jacobian of x, y, z with respect to u, v, w . (05 Marks)
 d. If $z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. (05 Marks)
- 5** a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$ and hence evaluate $\int_0^{\pi/2} \cos^6 x \, dx$ and $\int_0^{\pi/2} \cos^9 x \, dx$. (07 Marks)
 b. Evaluate $\int_0^1 \int_{x^2}^{\sqrt{x}} xy(x + y) \, dy \, dx$. (06 Marks)
 c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

- 6 a. Define Gamma and Beta functions. Show that $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$. (07 Marks)
- b. Prove that $\int_0^{\infty} x^2 e^{-x^4} dx \times \int_0^{\infty} e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. (07 Marks)
- c. Evaluate $\int_0^1 (\log x)^6 dx$. (06 Marks)
- 7 a. Solve the equation $\frac{dy}{dx} + x \tan(y - x) = 1$. (06 Marks)
- b. Solve $x^2 y dx - (x^3 + y^3) dy = 0$. (07 Marks)
- c. Solve $(e^y + y \cos xy) dx + (x e^y + x \cos xy) dy = 0$. (07 Marks)
- 8 a. Solve the equation $(D^3 + 1)y = 0$, where $D = \frac{d}{dx}$. (06 Marks)
- b. Solve the equation $(D^2 - 2D + 1)y = x e^x$. (07 Marks)
- c. Solve $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{2x} - \cos^2 x$. (07 Marks)
